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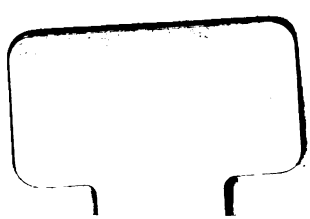

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MATHEMATICS

OF THE

PAPER LOCATION

OF A RAILROAD

BY

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The object of this article is to present the mathematical steps involved in preparing, from the paper location of a railroad, a set of mathematically consistent alinement notes by which to run the corresponding field location; and to suggest an orderly arrangement for the necessary computations.

Mr. W. B. Storey, Jr., in discussing a paper by Michael L. Lynch on Railroad Location, says:*

> On the Southern Pacific System the location is transferred from the paper to the ground, not by scaling, but by calculation. Each tangent is fixed, and the connecting curves are all calculated in the office by carrying the line from one fixed tangent around through the preliminary to the next tangent. These calculated notes are then given to a machine known as the locating party and put on the ground by it.

The writer does not remember having seen elsewhere any reference to the calculation of field notes; but in various articles on railroad location he has noted direct or implied reference to scaling.

To make clear what is meant by scaling and by calculating, and to show the advantage of calculating over scaling, let us find, by the two methods, the station and "plus" via the preliminary and via the location for the check point A, Fig. 1, the first crossing of the location and preliminary which have a common starting point, P_1.

BY SCALING.—The angle at P_1, between preliminary and location, is scaled in the course of preparing the notes for field location. Simply for the purpose of getting a check on the location at

*Trans. Am. Soc. C. E., Vol. XXXI., p. 92.

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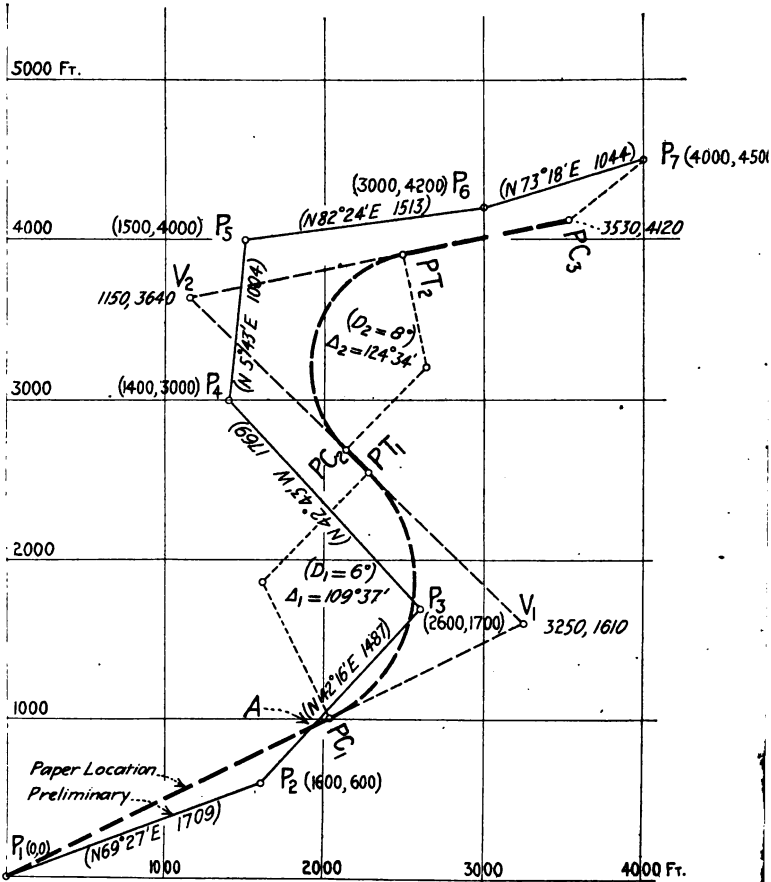

FIG. 1. METHOD OF PLOTTING PAPER LOCATION OF A RAILWAY LINE.

the point A we scale the distance P_1A on the location and the distance P_2A on the preliminary, thus obtaining the station and plus for A via each of the two lines. We now have of the triangle P_1P_2A the given side P_1P_2 (taken from the field notes of preliminary), the given angle P_1P_2A, the two scaled sides P_1A and P_2A, and one scaled angle, AP_1P_2. All the scaled quantities are affected by the errors of scaling, which are large compared with field errors, and it is evident that our values for the five parts of the triangle are mathematically inconsistent, since any three parts (which include one side) of a triangle determine the other parts. The result is that when the field party has located to A, and found they do not check on the preliminary within several feet, they have no means of telling how much of the error is due to surveying and how much to scaling. In this case the surveying checks the scaling, but there practically is no check on the field work.

BY CALCULATION.—By this method we scale only so many parts of the triangle P_1P_2A as will, together with the given part or parts, determine the size and shape of the triangle. Side P_1P_2 and angle P_1P_2A are known from the preliminary notes. If, then, we scale the angle P_2P_1A we shall have values for three parts of the triangle, from which we can compute the sides P_1A and P_2A. These five parts of the triangle—two given, one scaled and two computed—are mathematically consistent; and when the location party arrives at A and find they do not exactly check on the preliminary, they know that the error is all chargeable to field work (assuming that no errors have been made in computing). In this case the field work is really checked. Of course the error may be in running the preliminary or the location, or more probably in both. The check is on the surveying on both lines between P_1 and A. If the error is not within the limit of error permitted, the location is re-run (using the same notes), and if this is found to be practically correct, the next step is to re-run the preliminary from P_1 to A. If the error is not discovered here it must be found in the office work—computations or copying of notes. It cannot be due to scaling.

MATHEMATICS OF PAPER LOCATION: USING RECTANGULAR CO-ORDINATES.—The chief points of the preliminary, surveyed by any method or combination of methods, have been plotted by rectangular co-ordinates, and our map shows points P_1, P_2, P_3, P_4, P_5, P_6 and P_7. The co-ordinates of each point are written by the point, the value of the abscissa first. The engineer has drawn in pencil on the map the location P_1, PC_1, PT_1, PC_2, PT_2, PC_3. PC_1 and PT_1 are respectively the beginning and end of a 6-degree curve; and PC_2 and PT_2 are the beginning and end respectively of an 8-degree curve. PC_3 is the beginning of a curve, and is considered the end of this location. It will be noticed that each curve is numbered, and that each letter standing for a curve element is given for a subscript the number of its curve; e. g., the central angle of curve No. 2 is $\triangle_2$. All dimensions which appear on the map at this time are put in parentheses on our figure in order to distinguish them from the quantities computed in the work of getting out the location notes, which are written without parentheses. We have nothing to do here with the question of the proper place for the location line on the map: that is an engineering problem, while our work is to take the location line as we find it, and get out the notes for transferring it to the ground. Having no need of the topography in what follows, it has been omitted from our map. The distances have been computed here to the nearest foot only, although it is customary to consider tenths of a foot. Of course, in any given location the limit of error in field work should control the precision of computation. To save space there have been omitted from Fig. 1 the station numbering of the preliminary and of the location.

1. PRODUCE THE TANGENTS to intersect at V_1 and V_2.

2. SCALE THE CO-ORDINATES of the initial point, the final point, and of all the points of tangent intersection. In our case, the initial point of the location is at the origin of co-ordinates, and coincides with the initial point of the preliminary. The co-ordinates of V_1 and V_2 and PC_3 are written by the points on the map as they are scaled off,—x first and y second.

No further scaling is done, except that every computed distance and angle should be roughly checked by scaling. We are now ready to take up the computing.

3. COMPUTE THE BEARINGS AND LENGTHS OF TANGENTS PRODUCED TO INTERSECTION.—The plus direction of the y-axis is taken as north. The tangent of the bearing of P_1V_1 is

$$\tan \beta_{P_1V_1} = \frac{x_{V_1} - x_{P_1}}{y_{V_1} - y_{P_1}} = \frac{3{,}250 - 0}{1{,}610 - 0}$$

(Throughout, the subscript of an x or a y of any point is the symbol which designates that point; and the subscript of β is composed of the two symbols which designate the terminal points of a line.)

$$P_1V_1 = \frac{x_{V_1} - x_{P_1}}{\sin \beta_{P_1V_1}}$$

log 3,250 = 3.51188	log 3,250 = 3.51188
log 1,610 = 3.20683	log sin 63° 39′ = 9.95236
log tan 63° 39′ = 0.30505	log 3,627 = 3.55952

From the logarithmic computation we find the bearing and length of P_1V_1 to be N 63° 39′ E, 3,627 ft.

The tangent of the bearing of V_1V_2 is

$$\tan \beta_{V_1V_2} = \frac{x_{V_2} - x_{V_1}}{y_{V_2} - y_{V_1}} = \frac{1{,}150 - 3{,}250}{3{,}640 - 3{,}250} = \frac{-2{,}100}{2{,}030}$$

and

$$V_1V_2 = \frac{x_{V_2} - x_{V_1}}{\sin \beta_{V_1V_2}}$$

log — 2,100 = 3.32222 n	log 2,100 = 3.32222
log 2,030 = 3.30750	log sin 45° 58′ = 9.85669
45° 58′ = 0.01472 n	2,921 = 3.46553

That is, the bearing and length of V_1V_2 are N 45° 58′ W, 2,921 ft. (The negative value of the tangent shows that the bearing is either S E or N W, but the map indicates that the bearing is N W.)

The tangent of the bearing of V_2PC_3 is

$$\tan \beta_{V_2PC_3} = \frac{x_{PC_3} - x_{V_2}}{y_{PC_3} - y_{V_2}} = \frac{3,530 - 1,150}{4,120 - 3,640} = \frac{2,380}{480}$$

and

$$V_2PC_3 = \frac{x_{PC_3} - x_{V_2}}{\sin \beta_{V_2PC_3}}$$

log 2,380 = 3.37658	log 2,380 = 3.37658
log 480 = 2.68124	log sin 78° 36′ = 9.99135
log tan 78° 36′ = 0.69534	log 2,428 = 3.38523

giving for V_2PC_3 the bearing and length of N 78° 36′ E, 2,428 ft.

We now scale the map for a rough check on these computed bearings and distances.

4. COMPUTE CENTRAL ANGLES OF CURVES.—It is evident from the map that the central angle for curve No. 1 is

$$\triangle_1 = \beta_{P_1V_1} + \beta_{V_1V_2} = 63° 39′ + 45° 58′ = 109° 37′;$$

and the central angle for curve No. 2 is

$$\triangle_2 = \beta_{V_1V_2} + \beta_{V_2PC_3} = 45° 58′ + 78° 36′ = 124° 34′.$$

We roughly check these values by scaling the map with the protractor.

5. COMPUTE TANGENT-DISTANCES OF CURVES.—Using a "table of tangent-distances for a 1° curve" we find the tangent-distance for curve No. 1 is

$$T_1 = 1,355;$$

and for curve No. 2,

$$T_2 = 1,364.$$

We obtain a rough check on these by scaling.

If in any case the value of $\triangle$ is beyond the limits of the table, of course the tangent-distance must be computed from the formula:

$$T = R \tan \triangle/2.$$

6. COMPUTE CURVE LENGTHS.—Length of curve is equal to one hundred times the ratio of central angle to degree of curve.

Length of curve No. 1 is

$$L_1 = 100\ \triangle_1 / D_1 = 100\ (109° 37′) / 6° = 100\ (109.62) / 6 = 1,827.$$

Length of curve No. 2 is

$L_2 = 100\ \Delta_2 / D_2 = 100\ (124° 34') / 8° = 100\ (124.57) / 8 = 1{,}557.$

We now step off the curve lengths on the map to obtain a rough check on the computed values.

7. COMPUTE LENGTHS OF CENTER-LINE TANGENTS.—The length of the first tangent, as the map shows, is

$P_1PC_1 = P_1V_1 - T_1 = 3{,}627 - 1{,}355 = 2{,}272.$

The second tangent is

$PT_1\ PC_2 = V_1V_2 - (T_1 + T_2) = 2{,}921 - (1{,}355 + 1{,}364) = 202;$

And the third tangent is

$PT_2\ PC_3 = V_2\ PC_3 - T_2 = 2{,}428 - 1{,}364 = 1{,}064.$

The rough check by scaling is now employed.

8. COMPUTE STATION AND PLUS FOR CURVE AND TANGENT POINTS.—The station and plus for P_1 is 0 + 00.

The station and plus for PC_1 is $P_1PC_1 = (22 + 72)$.

The station and plus for PT_1 is $(22 + 72) + L_2 = (22 + 72) + 1{,}827 = (40 + 99)$.

The station and plus for PC_2 is $(40 + 99) + PT_1\ PC_2 = (40 + 99) + 202 = (43 + 01)$.

The station and plus for PT_2 is $(43 + 01) + L_3 = (43 + 01) + 1{,}557 = (58 + 58)$.

The station and plus for PC_3 is $(58 + 58) + PT_2\ PC_3 = (58 + 58) + 1{,}064 = (69 + 22)$.

Now we prick off the stations on the map, thus checking roughly these values.

9. COMPUTE ELEMENTS FOR CHECK POINTS.—On our map the location crosses the preliminary at the point A, which we use as a check point. There is no corresponding point at the other end of the line, and, in order to obtain a check, we draw the line PC_3P_7 and compute its bearing and length, to be run in the field as an auxiliary line to check on point P_7.

CHECK POINT A.—We first write the equations for lines, P_2P_3 and P_1V_1. The general equation for a straight line is $y = ax + b$,

where $a = (y_n - y_k) / (x_n - x_k)$

and $b = y_k - ax_k$.

(The subscripts k and n refer to the initial and final points respectively of any line.)

For line P_2P_3 $a = (y_3 - y_2) / (x_3 - x_2) = (1,700 - 600) (2,600 - 1,600) = 1,100 / 1,000 = 1.1$; and $b = y_2 - ax_2 = 600 - 1.1\ (1,600) = -1,160$. Equation of line P_2P_3 is therefore,

$$y = 1.1\ x + (-1,160)$$

or

$$y = 1.1\ x - 1,160.$$

For line P_1V_1

$$a = \frac{y_{V_1} - y_{P_1}}{x_{V_1} - x_{P_1}} = \frac{1,610 - 0}{3,250 - 0} = 0.4954$$

and

$$b = y_{P_1} - ax_{P_1} = 0 - a\,(0) = 0$$

making the equation of P_1V_1

$$y = 0.4954\ x.$$

We now compute the co-ordinates of the point A of intersection of P_2P_3 and P_1V_1.

If the equation of the first line be written

$$y = a\,x + b$$

and the equation of the second be written

$$y = a'\,x + b'$$

then the co-ordinates of this common point are:

$$x = \frac{b' - b}{a - a'}\ ;\ y = ax + b$$

and a check is had in the equation

$$y = a'x + b'$$

Substituting numerical values of a, b, a', b', for rou point A, we get:

$$x_A = \frac{0 - (-1,160)}{1.1 - 0.4954} = \frac{1,160}{0.6046} = 1,919$$

$$y_A = 1.1\,(1,919) + (-1,160) = 951$$

We scale the map to get a rough check on the computations.

It remains to find for A the station and plus via each line.

The distance

$$P_2A = \frac{x_A - x_2}{\sin \beta_{23}} = \frac{1,919 - 1,600}{\sin 42°16'} = \frac{319}{\sin 42°16'}$$

$$\log \quad 319 = 2.50379$$
$$\log \sin 42° 16' = 9.82775$$
$$\log \quad 474.8 = 2.67604$$

The logarithmic computation makes $P_2A = 474.8$.

The station and plus of A on the preliminary is

$$P_1 P_2 + P_2 A = 1{,}709 + 474.3 = 21 + 83.3.$$

The distance

$$P_1 A = \frac{x_A - x_{P_1}}{\sin \beta_{P_1 V_1}} = \frac{1{,}919 - 0}{\sin 63° 39'} = 2{,}141$$

as computed by logarithms here :

$$\log\ 1{,}919 = 3.28299$$
$$\log \sin 63° 39' = 9.95236$$
$$\log\ 2{,}141 = 3.33063$$

The station and plus for A on the location is, then, 21 + 41.

Scale the map to roughly check these values.

It is evident that station 21 + 41 on the location survey should coincide with station 21 + 83.3 on the preliminary.

In cases in practice a check point may be conveniently obtained by producing a location tangent to intersect the preliminary, and making the computation in the foregoing manner.

CHECK LINE PC_3P_7.—When a crossing of the location with the preliminary is not near at hand for a desired check on the field work, a check line, or tie line, is drawn between a chosen point of the location and a chosen point of the preliminary; and the bearing and length of the check line are computed. As an example, draw the check line PC_3P_7, and find the bearing by the equation:

$$\tan \beta_{PC_3 P_7} = \frac{x_{P_7} - x_{PC_3}}{y_{P_7} - y_{PC_3}} = \frac{4{,}000 - 3{,}530}{4{,}500 - 4{,}120} = \frac{470}{380}$$

and find the length

$$PC_3 P_7 = \frac{x_{P_7} - x_{PC_3}}{\sin \beta_{PC_3 P_7}}$$

$$\log\ 470 = 2.67210 \qquad \log\ 470 = 2.67210$$
$$\log\ 380 = 2.57978 \qquad \log \sin 51° 03' = 9.89081$$
$$\log \tan 51° 03' = 0.09232 \qquad \log\ 604.4 = 2.78129$$

We find the bearing and length of PC_3P_7 to be N 51° 03′ E, 604.4 ft. The location having been carried in the field to the point PC_3, the transitman deflects to the left at this point the angle $\beta_{V_2PC_3} - \beta_{PC_3P_7} = 78° 36' - 51° 03' = 27° 33'$, and the chainmen lay off 604.4 on this course, and

should by so doing arrive precisely at P_T on the preliminary.

10. COMPUTE CURVE DEFLECTIONS.—PC_1 is 22 + 72 making the first sub-chord on the 6-degree curve, 28 ft. The corresponding deflection is .28 x 3° = .28 x 180′ = 50′, i. e., the tangent deflection for station 23 is 50 minutes. The deflection for sub-chord at PC_2 is .99 x 240′ = 238′ = 3° 58′. The deflection for sub-chord at PT_1 is .99 x 180′ = 178′ = 2° 58′. The deflection for sub-chord at PT_2 is .58 x 240′ = 139′ = 2° 19′.

ALGORITHMS.—To facilitate explanation the foregoing computations have been put down in a

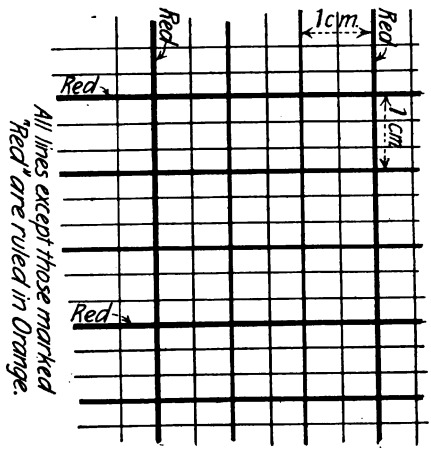

Fig. 2. Design for Rulings on Computing Paper.

rambling manner which makes the computed quantities hard to find when wanted. For a longer location than this, economy of time and effort requires that the routine computations be systematically arranged, that like operations as well as like quantities may be brought together. By this means entering the data and making the computations in a short time become largely mechanical processes, and at the end the computed quantities stand in tabular order and may be quickly found when wanted. For the computations of this ar-

ticle the following computation forms, or algorithms, are suggested. While these may not present the best arrangement, they will at least show the advantage of order over disorder. The reader may be interested in the fact that the writer's students buy for their computing, letter-size sheets of paper ruled on one side with the special design shown in Fig. 2. By the use of this paper any algorithm may be followed without drawing lines, and columns of digits are kept vertical automatically.

Algorithm 1 is for computing the bearing and length of lines which are terminated by points of known co-ordinates. Column A contains the numbers of the horizontal lines of the algorithm. Column B contains the symbols. x_k, y_k designate the co-ordinates of the initial point of any line; x_n, y_n designate the co-ordinates of the final point of the line; β_{kn} is the bearing, and d_{kn} is the length of the line. The lower half of this algorithm has to do with the check equation:

$$\text{hypot.} = (\text{base}^2 + \text{altitude}^2)^{1/2}.$$

Notice that the values on line 21 check those on line 12. Of course this check will not detect errors made in entering values of co-ordinates on lines 1, 2, 3, 4.

When a table of squares is at hand it may well be used in place of logarithms for the computation of checks. Columns C, D, E, F give for the line P_1 V_1, V_1 V_2, V_2 PC_2, and PC_2P_7 respectively, the numerical quantities corresponding to the symbols of Column B. It is suggested that the order of steps in computing be: (1) Enter the co-ordinates for all the lines concerned; (2) Set down on lines 5 and 6 the differences; (3) Enter all the logs on line 7; (4) Enter all the logs on line 8, and so on; every operation in Column C being immediately repeated for the succeeding columns.

Algorithm 2: Column A contains the numbers which have been used to designate the curves. In B will be found the degree of each curve. Each degree of curve is repeated in parenthesis, with the minutes expressed in decimal of a degree. In C are entered the central angles, each of which has been computed from two values of β taken from Algorithm 1. For example: C2 (i. e., the

ALGORITHM I.—Bearings and Distances on Location.

		Line P_1V_1	Line V_1V_2	Line V_2PC_3	Check Line PC_3P_7
1	x_k	0	3250	1150	3530
2	y_k	0	1610	4000	4120
3	x_n	3250	1150	3530	4000
4	y_n	1610	3640	4120	4500
5	$x_n - x_k$	3250	— 2100	2380	470
6	$y_n - y_k$	1610	2030	480	380
7	$\log (x_n - x_k)$	3.51188	3.32222 n	3.37658	2.67210
8	$\log (y_n - y_k)$	3.20683	3.30750	2.68124	2.57978
9	$\log \tan \beta_{kn}$	0.30505	0.01472 n	0.69534	0.09232
10	$\log \sin \beta_{kn}$	9.95236	9.85669	9.99135	9.89081
11	$\log d_{kn}$	3.55952	3.46553	3.38523	2.78129
12	d_{kn}	3627	2921	2428	604.4
13	β_{kn}	63° 39′	— 45° 58′	78° 36′	51° 08′
14	$\log (x_n - x_k)^2$	7.02376	6.64444	6.75316	5.34420
15	$\log (y_n - y_k)^2$	6.41366	6.61500	5.36248	5.15956
16	$(x_n - x_k)^2$	10560000	4410000	5664500	220900
17	$(y_n - y_k)^2$	2592000	4121000	230400	144400
18	$(x_n - x_k)^2 + (y_n - y_k)^2$	13152000	8531000	5894900	365300
19	$\log$ " "	7.11900	698100	6.77048	5.56265
20	$\frac{1}{2} \log$ " "	3.55950	3.46550	3.38524	2.78182
21	$[(x_n - x_k)^2 + (y_n - y_k)^2]^{1/2}$	3627	2921	2428	604.4
A	B	C	D	E	F

ALGORITHM II.—Curves and Tangents on Location.

	Curve No.	Degree of curve.	Central angle.	Tangent distance.	Length of curve.	Length of tangent produced to intersection.	1 tang. dist. or sum of 2 tangdists.	Length of tangent between curves.	Sta. and plus for PC.	Sta. and plus for PT.	Subchord deflection for PC.	Subchord deflection for PT.
		D°	△	T	L	d			PC	PT		
1.....	....	.....	.....	....	....	3627	1355	2272	.....	.....	.....	.....
2.....	1	6°00′	109°37′	1355	1827	....	....	....	22+72	40+99	0°50′	2°58′
3.....	....	(6.00)	(109°.62)	....	....	....	....	....	.....	.....	.....	.....
4.....	....	.....	.....	....	.....	2921	2719	202	.....	.....	.....	.....
5.....	2	8°00′	124°34	1364	1557	....	....	....	43+01	58+58	8°58′	2°19′
6.....	....	(8.00)	(124°.57)	....	....	....	....	.....	.....	.....	.....	.....
7.....	....	.....	.....	....	....	2428	1364	1064	.....	.....	.....	.....
8.....	3	.....	.....	....	....	....	....	....	69+22	.....	.....	.....
	A	B	C	D	E	F	G	H	I	J	K	L

quantity in Column C and on line (2) is 109° 37′, which is the sum of C13 (Alg. 1) and D13 (Alg. 1). C3 and C6 give the central angle with minutes reduced to decimal of a degree. D contains the tangent distance for each curve. E contains the curve lengths. In F are entered the lengths of tangents produced to intersection points, copied from line 12 of Alg. 1. G contains the amounts to be subtracted from quantities in F to obtain the quantities in H. For example: G4 is the sum of D2 and D5, and H4 is F4 less G4. I contains the station and plus for each point of curve; and J contains the station and plus for each point of

ALGORITHM III.—Line Equations.

Formulas: $a = (y_n - y_k) / (x_n - x_k)$; $b = y_k - ax_k$.

		Line P_2P_3	Line P_1V_1
1.....	x_k	1600	0
2.....	y_k	600	0
3.....	x_n	2600	3250
4.....	y_n	1700	1610
5.....	$y_n - y_k$	1100	1610
6.....	$x_n - x_k$	1000	3250
7.....	$\log (y_n - y_k)$	3.04139	3.20683
8.....	$\log (x_n - x_k)$	3.00000	3.51188
9.....	$\log a$	0.04139	9.69495
10.....	a	1.1	0.4954
11.....	$\log x_k$	3.20412	.....
12.....	$\log ax_k$	3.24551	.....
13.....	ax_k	1760	0
14.....	b	— 1160	0
15.....(Eq.)	$y = ax + b$	$y = 1.1x - 1160$	$y = 0.4954x + 0$
	A	B	C

ALGORITHM IV.—Co-ordinates of Intersection Point.

Formulas: $x = (b' - b) / (a - a')$; $y = ax + b$; check, $y = a'x + b'$.

			Check Point A.
1.....	(Eq.)	$y = ax + b$	$y = 1.1x - 1160$
2.....	(Eq.)′	$y = a'x + b'$	$y = 0.4954 + 0$
3.....		a	1.1
4.....		a'	0.4954
5.....		b	— 1160
6.....		b'	0
7.....		$b' - b$	1160
8.....		$a - a'$	0.6046
9.....		$\log (b - b)$	3.06446
10.....		$\log (a - a')$	9.78147
11.....		$\log x$	3.28299
12.....		$\log a$	0.04139
13.....		$\log a'$	9.69495
14.....		$\log ax$	3.32438
15.....		$\log a'x$	2.97794
16.....		ax	2110.
17.....		$a'x$	950.5
18.....		x	1919.
19.....		$y (= ax + b)$	950.
20.....		$y (= a'x + b')$	950.5

tangent. I2 is rewritten from H1; J2 is 12 plus E2; I5 is J2 plus H4; J5 is I5 plus E5, and so on. K and L contain the subchord deflections.

Algorithms 3 and 4 will need no explanation.

11.—ALINEMENT NOTES FOR LOCATION.—We are now ready to enter the results of our computations in the field book in the usual form of alinement notes.

Location Alinement Notes.

Sta.	Curve Point.	Deflections.	Computed Bearings.
69 + 22	PC_3	Check: Deflect 27° 33′ (bearing N 51° 03′ E) to left and run 604.′4 to P_7 which is Sta. 84 + 26 on preliminary.	
....			
....			
60			
59			N 78° 36′ E
58 + 58	PT_2	62° 17′	N 16° 20′ E
58		59° 58′	
57		55° 58′	
....		.....	
45		7° 58′	
44		3° 58′	
43 + 01	PC_2 8° R	0° 00′	N 45° 58′ W
43			
42			
41			N 45° 58′ W
40 + 99	PT_1	54° 48.5′	N 8° 51′ E
40		51° 50′	
39		48° 50′	
....		.. ....	
25		6° 50′	
24		3° 50′	
23		50′	
22 + 72	PC_1 6° L	0° 00′	N 63° 39 E
22			
21 + 41	Check: This point is 21 + 83.3 on preliminary.		
21			
....			
2			
1			
0	This is Sta. 0 of preliminary. For first location course deflect 5° 48′ to left of first preliminary course.		

THE MATHEMATICS OF PAPER LOCATION: USING POLAR CO-ORDINATES.—To compute the location field notes without employing rectangular co-ordinates, proceed thus:

1. The location having been laid down on the map as shown, scale P_1V_1, V_1V_2, V_2PC_3, and with the protractor scale the bearings of these three lines.

2. Compute the central angles $\triangle_1$ and $\triangle_2$.

3. Compute curve lengths L_1 and L_2.

4. Compute tangent distances T_1 and T_2.

5. Compute lengths of tangents P_1PC_1, PT_1 PC_2 and PT_2PC_3.

6. Write station and plus for P_1, PC_1, PT_1, PC_2, PT_2, and PC_3.

7. Find preliminary station and plus, and location station and plus, for check point A, thus: (a) Compute angles of triangle P_1P_2A; (b) From the known side and angles of this triangle compute the sides P_1A and P_2A; (c) The preliminary station and plus for A is $P_1P_2 + P_2A$; and the location station and plus for A is P_1A.

8. Having drawn on the map the check line PC_3P_7, compute the length and bearing of this line. This is done by treating PC_3P_7 as the "missing side" of the closed figure $AP_2P_4P_5P_6P_7PC_3V_2V_1A$.

9. Write the alinement notes in the field book.

The work of finding the "missing side" in Step 8, just above, involves practically all the operations required to compute the co-ordinates of the chief points of our map from the field notes of the preliminary survey.

Computing from the field notes the rectangular co-ordinates of the chief points of a survey is analogous to computing from the level notes the elevations of the stations on a profile, and has similar advantages.